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Nonlinear extended variational inequalities without differentiability: Applications and solution methods

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Abstract

We consider a class of nonlinear problems which is intermediate between equilibrium and variational inequality ones. Several classes of applications of such problems are described. Iterative methods are proposed for finding a solution. The methods are utilized without differentiability properties of the mappings and converge to a solution under weakened monotonicity type assumptions.

Keywords: Extended variational inequality; Non-differentiable mappings; Combined relaxation method; Equilibrium problems; Composite optimization; Descent method

1. Introduction

Most equilibrium type problems arising in applications are usually formulated as either general equilibrium problems or variational inequalities. Although the theory and solution methods for these problems have been investigated rather well (see e.g. [1–4]), the methods taking into account peculiarities of some classes of such problems may appear more efficient in comparison with the general methods without special adjustment techniques.

In this paper, we consider the following class of *nonlinear extended variational inequality problems* (NEVI for short): Find a point $x^* \in K$ such that

$$\langle G(x^*), H(x) - H(x^*)\rangle \geq 0 \quad \forall x \in K, \tag{1}$$

where K is a convex set in the Euclidean space $\mathbb{R}^n$, $G : K \to \mathbb{R}^m$ and $H : K \to \mathbb{R}^m$ are given continuous mappings. Of course, if H is the identity map I, then $m = n$ and (1) reduces to the usual variational inequality (VI for short). At the same time, problem (1) can be viewed as a particular case of the general *equilibrium problem* (EP for short): Find $x^* \in K$ such that

$$\Phi(x^*, y) \geq 0 \quad \forall y \in K, \tag{2}$$

if we set

$$\Phi(x, y) = \langle G(x), H(y) - H(x)\rangle. \tag{3}$$

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